Analytical results for the statistics of the ideal Bose gas in different statistical ensembles

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Although the statistics of an ideal Bose gas was studied in earlier works [1–3], its description in the microcanonical ensemble (MCE) remains difficult. Recent experiments highlight the usefulness of the MCE and emphasize the importance of choosing an appropriate statistical ensemble for the theoretical description. We derive analytical formulas for fluctuations and all higher moments of the condensate population in the MCE and in the canonical ensemble (CE). Our work is based on the saddle-point method introduced by Holthaus [1], which we refine to obtain results at arbitrary levels of approximation for any physically realizable non-interacting system. The ratio of the maxima of fluctuations in the MCE and CE in a 3D harmonic trap converges very slowly to approximately 0.39 in the thermodynamic limit, confirming the result of Z. Idziaszek [4].

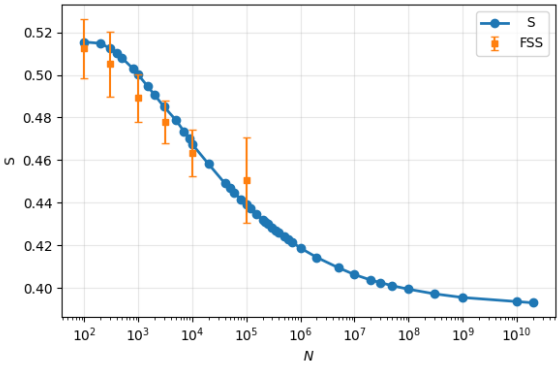

FIG. 1. Ratio of the maximal condensate fluctuations in MCE and CE $S = \max\{\delta^2 n_0^{(\mathrm{MCE})}\}/\max\{\delta^2 n_0^{(\mathrm{CE})}\}$ for BEC in a 3D harmonic trap, as a function of the particle number N. Blue circles denote the results obtained from analytical formulas, while orange squares show Fock State Sampling Method (FSS) estimates [5]. The ratio converges slowly to ≈ 0.39 in the thermodynamic limit.

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